



Editorial

A Paraconsistent Decagon

A scientific meeting is a success when it promotes a blend of ideas, proposals, guidelines and criticisms that will influence further research in the area. Measured by such standards, the *Workshop on Paraconsistent Logic (WoPaLo)*,¹ was a very successful meeting. The present volume intends to attest this, bringing a finely revised collection of articles, a sequel of a previous electronic publication containing all the extended abstracts available under the title “Proceedings of the Workshop on Paraconsistent Logic (WoPaLo)” at **CLE e-Prints** Vol. 2(7), 2002.²

Paraconsistency, in very simple terms, is the logical study of inconsistent yet non-trivial theories or premise sets. It is a sign of maturity of contemporary logic that logicians were liberated from the *horror contradictionis* and started to build theories in which contradictions may not only subsist but in fact be the object of mathematical study. In brief terms, paraconsistency is *neither* the sport of collecting contradictions or passion of cultivating them, *nor* the craft of building eccentric logics, but rather the delicate art of carving logical systems with less material than in the abundance and security of classical territory, where contradictions are banned and one has at her disposal a surplus of building material.

Without any intention of historical completeness, but just to mark an area of research, it may be appropriate to recall here the names of the Polish logician Stanisław Jaśkowski and the Brazilian logician Newton C.A. da Costa, who independently urged for the development of logical systems in which the classical principle that demands anything to follow from a contradiction would be logically controlled. Much of the impetus of the research on paraconsistent logics is motivated by the attainment of those goals: Taming the explosive power of contradictions, and yet being able to deliver logics sharing with the classical paradigm many desirable features, as model existence results and model theory, reasonable proof methods, algebraic and set-theoretic counterparts, (un)decidability and so on. More-

¹ Held in Trento, Italy, from 5 to 9 August 2002, as part of the XIV *European Summer School in Logic, Language and Information* (ESSLLI 2002), organized under the auspices of the European Association for Logic, Language and Information (FoLLI). For more information, check the WoPaLo homepage: <http://logica.ugent.be/WoPaLo>.

² Check http://www.cle.unicamp.br/e-prints/abstract_16.html.

over, as paraconsistent logics necessarily deviate from classical logic, specific properties have to be demonstrated in justification of the particular approach.

The significance of paraconsistency from the purely logical point of view is founded in the meticulous dissociation between the general notions of contradictoriness and of triviality of a theory. So, on the one hand, if explosiveness due to contradictions is controlled, there is room for logics that allow for contradictory but non-trivial theories (even if these are considered as temporary, or as restricted to certain circumstances). On the other hand, the challenge is to construct logics keeping with such dissociation.

One can identify at least three main definitions of paraconsistent logic in the already vast literature of the area. Jaśkowski's definition proposes the investigation of logics that could serve as bases for non-explosive theories (i.e. theories which would not collapse into overcompleteness in the presence of contradictions). Da Costa's definition calls for the investigation of logics that could support contradictory theories while at the same time preventing some of those theories from being trivial. A third definition considers that consistent logics are those which are both explosive and non-trivial, and calls for the investigation of paraconsistent logics exactly as those which are inconsistent yet still non-trivial. The equivalence of the three definitions can be shown under appropriate qualifications about the properties of the underlying inference relations of the considered logics.

It comes as no surprise that a mathematically well-founded theory that diminishes the gulf between pragmatical reasoning and symbolic reasoning, with its effects on argumentation, on thinking and even on constructing mathematical proofs (one cannot so candidly blame contradictions for mathematical existence or non-existence any more) forces a reexamination of the classical account of rationality. Induction, deduction and abduction cannot ignore this new frontier. The science of information, with its twilight zone of incongruity and vagueness, claims for the investigation of methods helping to tolerate and even to take profit of contradictions, be they *de dicto* or *de facto*. The possibility of paraconsistent reasoning has thus matched the interest of philosophers, linguists and computer scientists. Interesting dialectical and relevance logics are paraconsistent, and some concepts in the philosophy of science can be recast in the light of paraconsistency. Some kinds of linear and quantum logics can be shown to benefit from the paraconsistent viewpoint. The same holds for software engineering, database theory, model checking, theorem proving, logic programming, data mining, evolutionary computation, semantic web and model-based reasoning, fields that are quickly improving their standards to work under logics that lose little from the classical point of view, and yet permit to learn from contradictions.

On what this collection is made of

The collection of papers in the present collection makes justice to the description of paraconsistency we have sketched above. We include a brief invitation to each paper, with the sole intention of offering an appetizing preview of what is coming. Of course, the invitation will never make justice to reading the full papers: We can only hope the reader will accept it.

“Paraconsistent logic from a modal viewpoint”, by Jean-Yves **Béziau**, offers a modal perspective on paraconsistent negation. By introducing a new approach based on the well-

known modal logic **S5** and a novel four-valued logic with modal flavor, he capitalizes on a new look of the (modal) square of oppositions, pointing out that one of the corners of the square has no denotation in natural language, and arguing that this nameless corner has to do with paraconsistent negation.

In “Models for a paraconsistent set theory”, Thierry **Libert** searches for natural models for paraconsistent extensions of positive set theory. In order to circumvent traditional difficulties such as Russell’s paradox, paraconsistent sets are actualized by considering membership and non-membership as somewhat independent properties. In reviewing previous related approaches, some emphasis is put on the use of fixed-point arguments and the distinction abstraction/comprehension in the formalization of set theory. The existence of natural topological models for comprehension is then established.

Sergei P. **Odintsov**, in “On the structure of paraconsistent extensions of Johansson’s logic”, gives a compact presentation of basic results on the class **Jhn** of non-trivial paraconsistent yet partially explosive extensions of minimal (or Johansson’s) logic. The author argues that, unlike the class of intermediate logics, the class **Jhn** has an interesting and non-trivial global structure. The study of this class is based on an adequate presentation of algebraic and Kripke-style semantics for extensions of minimal logic.

In “An encompassing framework for paraconsistent logic programs”, João **Alcântara**, Carlos Viegas **Damásio** and Luís Moniz **Pereira** propose a framework that extends anti-tonic logic programs and illustrate the use of bilattices in logic programming for reasoning with uncertain, incomplete and inconsistent information, in a way that immediately appeals to those who are familiar with the work of M. Fitting and with probabilistic deductive databases. As motivation, several interesting examples of the large range of applications of the authors’ techniques are contemplated. The framework provided is argued to be strong enough as to allow for the embedding of other logic programming systems.

“Paraconsistent informational logic”, by Paola **Forcheri** and Paolo **Gentilini** stresses an application of paraconsistent logics to formal epistemology. The authors present a formalism to express conjectures as formal objects, where the deductive apparatus of conjecturing agents is conflated with some given environment system. In such an interaction of agents with environment, inconsistencies might quite reasonably emerge. According to a logical entropy measure they introduce, a theory which contradicts another one can still constitute a very good conjecture with respect to the latter, and this clearly extends the notion of rationality of an agent holding such a theory. The formalism is then applied for a particular logic of formal inconsistency, and a proof-theoretical investigation follows.

In “Aristotle’s Thesis between paraconsistency and modalization” Claudio **Pizzi** revisits some paraconsistent extensions of relevant logics in relation to Aristotle’s thesis $\neg(A \rightarrow \neg A)$ and the Law of Simplification $(A \wedge B) \rightarrow B$. The paper aims to show that interpretations of Aristotle’s Thesis may vary according to different understandings of the arrow \rightarrow , while the role of paraconsistency is discussed therein. It is proved that under a certain definition of the arrow, Aristotle’s Thesis subjoined to the minimal normal system **K** yields a system equivalent to the deontic system **KD**, and this modalization allows interpreting the arrow as the expression of relevance in a specific modal sense.

In “Combining classical logic, paraconsistency and relevance”, Arnon **Avron** explores relevance concerns inside paraconsistent domains and presents a logic having a simple semantics and a cut-free Gentzen-type proof system. This logic combines classical logic

with relevance logics and paraconsistent logics in da Costa's style. Besides obtaining some interesting properties, the author defends the qualities of the underlying language of his logic from the semantical viewpoint.

In "Anti-intuitionism and paraconsistency" Andreas **Brunner** and Walter A. **Carnielli** carefully discuss and investigate the question of the duality between intuitionistic and paraconsistent thought paradigms. They argue that the quest for duality requires a multiple-conclusion logical environment where refutative systems must be considered, as well as the usual demonstrative systems. By proposing new hierarchies of anti-intuitionistic logics (that include formulations of a dual-Johánsson's logic and of Gödel's three-valued logic) and studying their relations with intuitionistic logics, they show that all anti-intuitionistic logics are paraconsistent, but those particular duals give rise to brand new paraconsistent logics. On the other hand, duals of paracomplete (or maximal weakly intuitionistic) many-valued logics do coincide with well-known many-valued paraconsistent logics.

In "On negation: pure local rules" João **Marcos** carries out a systematic study of the properties of negation from the point of view of abstract deductive systems. By means of a unifying framework of multiple-conclusion consequence relations, several rules for negation, among them the generalized forms of proof by cases, of *consequentia mirabilis* and of *reductio ad absurdum* are expressed. Moreover, dualization reveals many rules heretofore unspoken of. This framework also permits careful definitions of varieties of paraconsistency and of the dual paracompleteness, allowing for *pseudo-scotus* and *ex contradictione* to be distinguished and for a comprehensive version of the Principle of Non-Triviality to be expressed. The author also inaugurates the study of logics and of logical constants from a negative perspective, and supports the claim that negative rules are in a sense more fundamental than positive rules. A survey of the related literature on negation is advanced, and many corrections or updates are made on proposals and results by other authors.

In "A procedural criterion for final derivability in inconsistency-adaptive logics", Diderik **Batens** investigates a goal-directed proof procedure for the inconsistency-adaptive logic **ACLuN1**. Adaptive logics characterize inference relations that lack a positive test, which occur in ordinary reasoning and scientific reasoning processes. The proofs of adaptive logics are necessarily dynamic, whence it is important to decide whether a formula derived at a stage is 'finally derived'. The proposed procedure defines an algorithm for final derivability at the propositional level that can be extended to the predicative level, providing a criterion for final derivability there. The procedure is generalizable to all flat adaptive logics.

We believe that the ten fully refereed and well-worked papers published here provide a nice and informative portrait, quite representative of the strength and of the variety in the field of paraconsistent logics. In composing this collection we received help from an impressive number of instances and people to whom we wish to express our thanks: To the Organizing Committee of the ESSLLI 2002 for all the help and partial financial support provided to the WoPaLo and its organizers, to Dov Gabbay for his invitation to publish this material in the *Journal of Applied Logic*, to Mrs. Jane Spurr for her assistance in materializing this possibility, and specially to the inestimable help of a(n otherwise anonymous) team of referees who assisted us in selecting the submissions, improving and correcting mistakes, adding references, and decisively contributed to the quality of this volume. We name them here as a sign of our gratitude: Martin Allen, Ofer Arieli, Peter Arnd, Arnon

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